Higher memory effects

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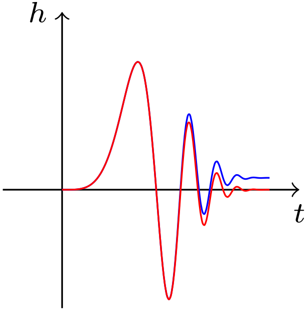
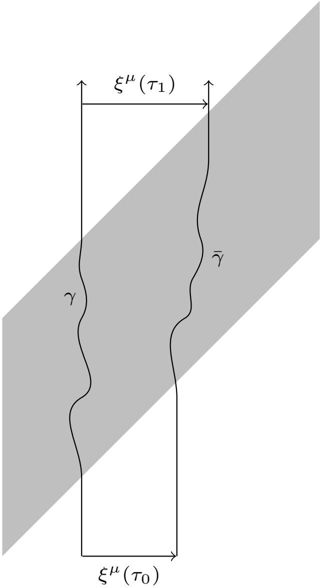
Memory effect

- Gravitational wave (displacement) memory: change in separation of initially comoving, freely falling observers [Zel'dovich & Polnarev, 1974]
- Appears as DC offset for a GW detector $\implies$ *fundamentally different* type of signal!





Origin of the memory

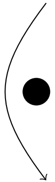
- Quadrupole formula:

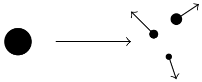
$$h_{ij} = \frac{2\ddot{Q}_{ij}}{r} + O(1/r^2) \implies \text{when is } \Delta\ddot{Q}_{ij} \neq 0?$$

- $Q_{ij} \sim m x_i x_j$ & $\ddot{x}_i = 0$ at late times, so

$$\Delta\ddot{Q}_{ij} \sim m\Delta\left[v_i v_j\right] \implies \text{when } v_i \text{ changes!}$$

- Note: applies to *unbound systems* w/ particles flying off to infinity





Memories and "strong gravity"

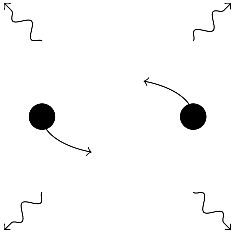
- Bound systems: "particles flying off to infinity" are *gravitational waves*

- Source is *quadratic* in the field:

$$T_{\alpha\beta} \sim \langle \dot{h}_{\alpha\gamma} \dot{h}^{\gamma}{}_{\beta} \rangle \implies \Delta h(u) \sim r \int_{-\infty}^{u} |\dot{h}|^2$$

(with $h \equiv h_+ - i h_\times \sim 1/r$)

- Probes nonlinearity in *propagation* regime



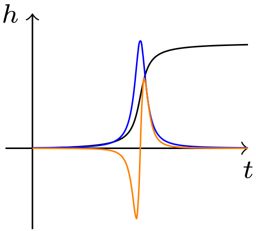
[Christodoulou, 1991], [Blanchet & Damour, 1992 (1990)], [Thorne, 1992]

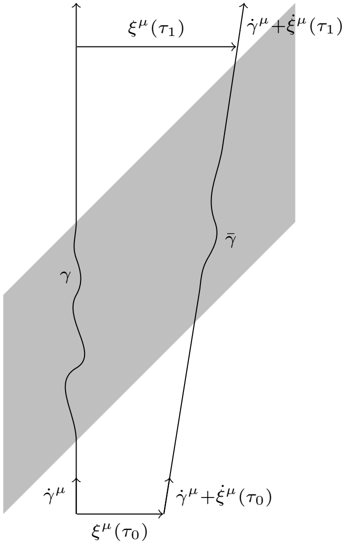
Higher memories

- Two "simple" generalizations:
  - Relax assumptions: initially comoving or freely falling
  - Consider other properties (e.g., final relative velocity)
- Additional, memory-like effects (drift = spin + c.o.m.):

$$\begin{pmatrix} \boldsymbol{\xi}_f \\ \dot{\boldsymbol{\xi}}_f \end{pmatrix} = \begin{pmatrix} \boldsymbol{\xi}_i + \Delta\tau \dot{\boldsymbol{\xi}}_i \\ \dot{\boldsymbol{\xi}}_i \end{pmatrix} + \begin{pmatrix} \text{displacement} & \text{drift} \\ \text{velocity} & \text{kick} \end{pmatrix} \begin{pmatrix} \boldsymbol{\xi}_i \\ \dot{\boldsymbol{\xi}}_i \end{pmatrix}$$
$$+ (\cdots) + \begin{bmatrix} \text{“higher memories”} \\ \text{(from acceleration)} \end{bmatrix}$$

- Appear as different types of non-oscillatory GW features
- Probe "subleading" nonlinearities in propagation of gravitational waves





Outline

I. Definition from observables

II. Asymptotically flat spacetimes

III. Applications to binary inspirals

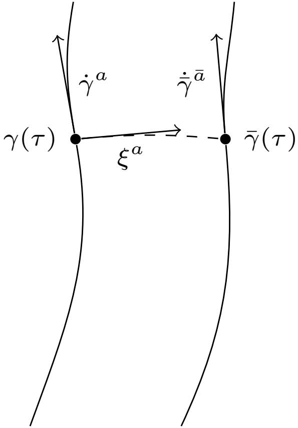
IV. Conclusions and future work

Not in this talk: detectability (except one slide), symmetries, soft theorems…

Geodesic deviation

- Two observers, following γ and $\bar{\gamma}$, w/ four-velocities $\dot{\gamma}^a$ and $\dot{\bar{\gamma}}^{\bar{a}}$
- Separation vector ξ^a tangent to unique geodesic between $\gamma(\tau)$ and $\bar{\gamma}(\tau)$
- Geodesic deviation equation:

$$\ddot{\xi}^a = -\underbrace{R^a{}_{cbd}\dot{\gamma}^c\dot{\gamma}^d}_{\equiv R^a{}_{\dot{\gamma}b\dot{\gamma}}}\xi^b + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}})^2$$



For an excellent review, see [Vines, 2014]

General solution

- Recall: cannot add/subtract tensors at different points $\implies$ cannot solve *tensor* ODEs
- Parallel-transported tetrad $\{\boldsymbol{e}_\alpha\}$ $\implies$ *scalar* ODE:

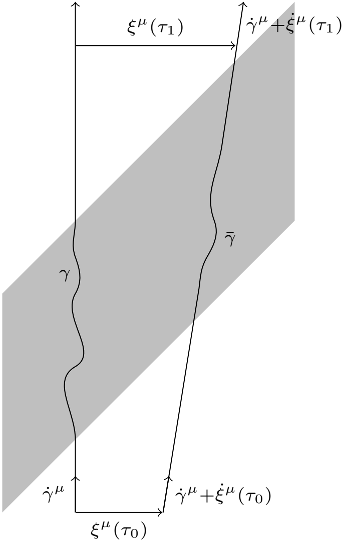
$$\frac{\mathrm{D}\boldsymbol{e}_\alpha}{\mathrm{d}\tau} = 0 \implies \ddot{\xi}^\alpha(\tau) = -R^\alpha{}_{\dot\gamma\beta\dot\gamma}(\tau)\xi^\beta(\tau) + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}})^2$$

- General solution (linear order):

$$\xi^\mu(\tau') = A^\mu{}_\nu(\tau',\tau)\xi^\nu(\tau) + B^\mu{}_\nu(\tau',\tau)\dot{\xi}^\nu(\tau)$$

where $\boldsymbol{A}, \boldsymbol{B}$ solve (w/ appropriate BC's)

$$\partial^2_{\tau'} U^\mu{}_\nu(\tau',\tau) = -R^\mu{}_{\dot\gamma\rho\dot\gamma}(\tau')U^\rho{}_\nu(\tau',\tau)$$



"Geodesic" memory effects

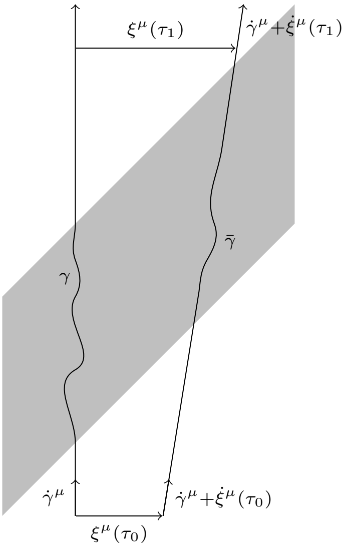
- Matrix form of solution:

$$\begin{bmatrix} \boldsymbol{\xi}(\tau') \\ \dot{\boldsymbol{\xi}}(\tau') \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}(\tau',\tau) & \boldsymbol{B}(\tau',\tau) \\ \partial_{\tau'}\boldsymbol{A}(\tau',\tau) & \partial_{\tau'}\boldsymbol{B}(\tau',\tau) \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}(\tau) \\ \dot{\boldsymbol{\xi}}(\tau) \end{bmatrix}$$

- Comparing to memory effects:

$$\begin{pmatrix} \text{displacement} & \text{drift} \\ \text{velocity} & \text{kick} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Delta A} & \boldsymbol{\Delta B} \\ \partial_{\tau'}\boldsymbol{\Delta A} & \partial_{\tau'}\boldsymbol{\Delta B} \end{pmatrix}$$

$$\text{where} \qquad \begin{cases} \boldsymbol{\Delta A}(\tau',\tau) = \boldsymbol{A}(\tau',\tau) - \mathbb{I}, \\ \boldsymbol{\Delta B}(\tau',\tau) = \boldsymbol{B}(\tau',\tau) - \Delta\tau\mathbb{I} \end{cases}$$



The addition of acceleration: "Non-geodesic" deviation

- If $\bar{\gamma}$ accelerated, geodesic deviation modified:

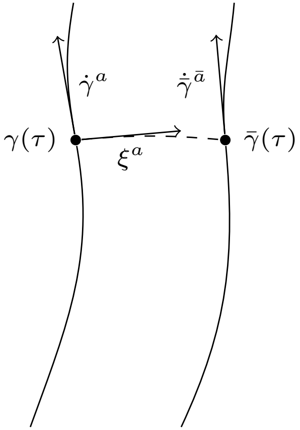
$$\ddot{\xi}^a = -R^a{}_{\dot{\gamma}b\dot{\gamma}}\xi^b + \overbrace{\underbrace{g^a{}_{\bar{a}}}_{\text{parallel transport map}}\ddot{\bar{\gamma}}^{\bar{a}}}^{\text{“relative acceleration” } a^a} + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, \boldsymbol{a})^2,$$

- Solution on our tetrad (linear order):

$$\ddot{\xi}^\alpha(\tau) = -R^\alpha{}_{\dot{\gamma}\beta\dot{\gamma}}(\tau)\xi^\beta(\tau) + a^\alpha(\tau)$$

$$\Downarrow$$

$$\xi^\alpha(\tau') = A^\alpha{}_\beta(\tau',\tau)\xi^\beta(\tau) + B^\alpha{}_\beta(\tau',\tau)\dot{\xi}^\beta(\tau) + \int_\tau^{\tau'} \mathrm{d}\tau''\, B^\alpha{}_\beta(\tau',\tau'')a^\beta(\tau'')$$



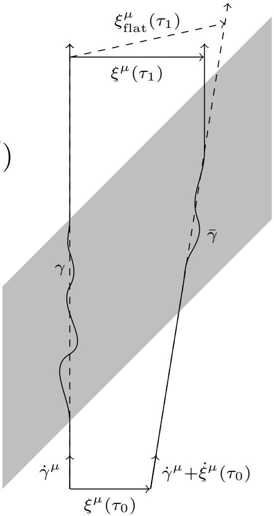
Curve deviation

- Part of solution is an uninteresting, "kinematic" piece:

$$\ddot{\xi}^\alpha_{\text{flat}}(\tau) = a^\alpha(\tau) \implies \begin{cases} \xi^\alpha_{\text{flat}}(\tau') = \xi^\alpha(\tau) + (\tau' - \tau)\dot{\xi}^\alpha(\tau) \\ \qquad\qquad + \int_\tau^{\tau'} \mathrm{d}\tau''(\tau' - \tau'')a^\alpha(\tau'') \end{cases}$$

- Subtracting off yields *curve deviation* observable:

$$\Delta\xi^\alpha(\tau', \tau) \equiv \xi^\alpha(\tau') - \xi^\alpha_{\text{flat}}(\tau')$$
$$= \overbrace{\Delta A^\alpha{}_\beta(\tau', \tau)}^{\text{displacement}}\, \xi^\beta(\tau) + \overbrace{\Delta B^\alpha{}_\beta(\tau', \tau)}^{\text{drift}}\, \dot{\xi}^\beta(\tau)$$
$$\underbrace{+ \int_\tau^{\tau'} \mathrm{d}\tau''\, \Delta B^\alpha{}_\beta(\tau', \tau'')a^\beta(\tau'')}_{\text{"higher memories"}}$$



"Unification" of higher memories [**Grant**, 2401.00047]

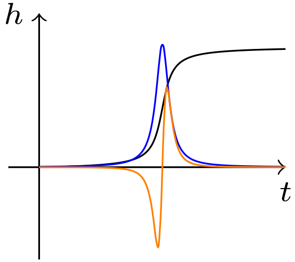
- Higher memories characterized by:

$$\underbrace{\Delta \underset{(n)}{\alpha}{}^{\alpha}{}_{\beta}(\tau', \tau)}_{\text{dependence on initial acceleration, jerk, etc.}} \equiv \frac{1}{n!}\int_{\tau}^{\tau'} \mathrm{d}\tau'' \, (\tau'' - \tau)^n \Delta B^{\alpha}{}_{\beta}(\tau', \tau'')$$

- Identities involving $\partial_\tau A^{\alpha}{}_{\beta}(\tau', \tau), \partial_\tau B^{\alpha}{}_{\beta}(\tau', \tau) \implies$

$$\underbrace{\underset{(n)}{\mathscr{E}}{}^{\alpha}{}_{\beta}(\tau', \tau)}_{\text{"moments"}} = \begin{cases} \Delta A^{\alpha}{}_{\beta}(\tau', \tau) & n = 0 \\ \Delta B^{\alpha}{}_{\beta}(\tau', \tau) & n = 1 \\ \Delta \underset{(n-2)}{\alpha}{}^{\alpha}{}_{\beta}(\tau', \tau) & n \geq 2 \end{cases}$$

$$= -\frac{1}{n!}\int_{\tau}^{\tau'} \mathrm{d}\tau''(\tau'' - \tau)^n \underbrace{B^{\alpha}{}_{\mu}(\tau', \tau'') R^{\mu}{}_{\dot\gamma\beta\dot\gamma}(\tau'')}_{\text{only piece needed for computation}}$$



Outline

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II. Asymptotically flat spacetimes

III. Applications to binary inspirals

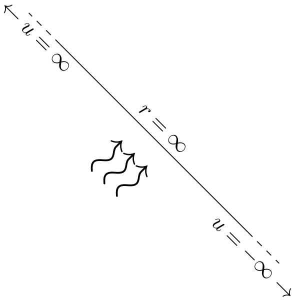
IV. Conclusions and future work

Bondi(-Sachs) coordinates

$$g_{uu} = -1 + \frac{2m}{r} + O(1/r^2), \qquad g_{ur} = -1 + O(1/r^2)$$

$$g_{ui} = -\frac{1}{2}\mathscr{D}^j C_{ij} + \frac{1}{r}\left(\frac{2}{3}N_i + \cdots\right) + O(1/r^2),$$

$$g_{ij} = r^2\left\{[1 + O(1/r^2)]h_{ij} + \frac{1}{r}\left(C_{ij} + \frac{1}{r^2}\sum_{n=0}^{\infty}\frac{1}{r^n}\underset{(n)}{\mathcal{E}}{}_{ij}\right)\right\}$$



- h_{ij}, $\mathscr{D}_i$ metric & connection on sphere
- *Shear* C_{ij} (waveform), “higher Bondi aspects” $\underset{(n)}{\mathcal{E}}{}_{ij}$
- m, N^i: *mass* and *angular momentum aspect*
- $N_{ij} = \partial_u C_{ij}$: *news*, indicates presence of radiation
- m, N^i, $\underset{(n)}{\mathcal{E}}{}_{ij}$: properties of source (essentially $\mathrm{Re}[\psi_2]$, ψ_1, ψ_0^n)

Asymptotic form of curve deviation

- Curvature at leading order:

$$R^i{}_{uju} = -\frac{1}{2r}\partial_u N^i{}_j + O(1/r^2)$$

- For asymptotic observers w/ $\dot{\gamma}^a = (\partial_u)^a + O(1/r)$, moments given by:

$$\underset{(n)}{\mathscr{E}}{}^i{}_j(u', u) = \frac{1}{2r}\left[(n+1)\underset{(n)}{\mathcal{N}}{}^i{}_j(u', u; u) - (u' - u)\begin{cases} 0 & n = 0 \\ \underset{(n-1)}{\mathcal{N}}{}^i{}_j(u', u; u) & n > 0 \end{cases}\right]$$
$$+ O(1/r^2)$$

where "moments of news" given by

$$\underset{(n)}{\mathcal{N}}{}^i{}_j(u, u'; \tilde{u}) \equiv \frac{1}{n!}\int_u^{u'} \mathrm{d}u''\,(u'' - \tilde{u})^n N^i{}_j(u'')$$

Two useful notions of moments

$$\underset{(n)}{\mathcal{N}}{}^{i}{}_{j}(u', u; \tilde{u}) \equiv \frac{1}{n!} \int_{u}^{u'} \mathrm{d}u'' \, (u'' - \tilde{u})^{n} N^{i}{}_{j}(u'')$$

"Mellin" moments (original approach)

- Use $\tilde{u} = u$:

$$\underset{(n)}{\widetilde{\mathcal{N}}}{}^{i}{}_{j}(u', u) \equiv \underset{(n)}{\mathcal{N}}{}^{i}{}_{j}(u', u; u)$$

- Directly related to observables
- For $u = 0$, $u' = \infty$, related to *Mellin transform:*

$$\mathcal{M}\{f\}(n) \equiv \int_{0}^{\infty} u^{n-1} f(u) \mathrm{d}u$$

"Cauchy" moments (this talk)

- Use $\tilde{u} = u'$, with a sign change:

$$\underset{(n)}{\mathcal{N}}{}^{i}{}_{j}(u', u) \equiv (-1)^{n} \underset{(n)}{\mathcal{N}}{}^{i}{}_{j}(u', u; u')$$

- Related to *Cauchy's formula for repeated integration:*

$$\underset{(n)}{\mathcal{N}}{}^{i}{}_{j}(u', u) = \int_{u}^{u'} \mathrm{d}u'' \underset{(n-1)}{\mathcal{N}}{}^{i}{}_{j}(u'', u)$$

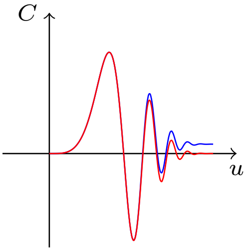
Computation of the moments

- Given a full waveform C_{ij}, one can easily compute these moments by integration

- Unfortunately, we do not have a "full" waveform:
  - Any approximation scheme (PN, self-force, etc.) only valid up to some order
  - In NR one often *extrapolates* signals at finite radius to infinity (CCE has mostly fixed this)

  $\Longrightarrow$ Moments computed directly are inaccurate!

- Fortunately, relationships exist b/w these moments and C_{ij}, m, N^i, $\underset{(n)}{\mathcal{E}}{}_{ij}$, providing
  - Consistency checks for "exact" waveforms
  - Corrections to approximate waveforms



Schematic form of evolution equations

- Define "electric" metric functions:

$$Q_0 \equiv m, \qquad Q_1 \equiv \mathscr{D}_i N^i, \qquad Q_{n+2} \equiv \mathscr{D}^i \mathscr{D}^j \underset{(n)}{\mathcal{E}}{}_{ij}$$

(omit magnetic versions for simplicity)

- Evolution equations take the following form:

$$\dot{Q}_0 = \frac{1}{4} \mathscr{D}^i \mathscr{D}^j N_{ij} - \mathcal{F}_0, \qquad \dot{Q}_n = \mathcal{D}_n Q_{n-1} - \mathcal{F}_n - \mathcal{G}_n$$

where

  - $\mathcal{D}_n$: differential operator on sphere
  - $\mathcal{F}_n$: nonlinear "flux" term, depends on N_{ij} (vanishes in nonradiative regions)
  - $\mathcal{G}_n$: nonlinear "pseudoflux" term, does *not* vanish in nonradiative regions (only exists for $n \geq 2$)

Construction of “charges”

$$\dot{Q}_0 = \frac{1}{4}\mathscr{D}^i\mathscr{D}^j N_{ij} - \mathcal{F}_0, \qquad \dot{Q}_n = \mathcal{D}_n Q_{n-1} - \mathcal{F}_n - \mathcal{G}_n$$

- Note: $\dot{Q}_0$ vanishes when $N_{ij} = 0$; call such quantities “charges”
- $Q_{n\geq 1}$ *not* charges, as $Q_{n-1} \neq 0$ when $N_{ij} = 0$ (and $\mathcal{G}_n \neq 0$ when $N_{ij} = 0$ for $n \geq 2$)
- Can be modified to form charges, however (*not* unique!):

$$\tilde{Q}_n(u;\tilde{u}) \equiv Q_n(u) + \sum_{m=0}^{n-1} \frac{(\tilde{u}-u)^{n-m}}{(n-m)!}\mathcal{D}_n \cdots \mathcal{D}_{m+1} Q_m(u) + \underbrace{(\cdots)}_{\text{constructed from } \mathcal{G}_{2\leq m\leq n}}$$

Relationship to moments

- Zeroth moment: integrate Q_0 evolution equation:

$$\frac{1}{4}\mathscr{D}^i\mathscr{D}^j \underset{(0)}{\mathcal{N}}_{ij}(u',u) = Q_0(u') - Q_0(u) + \int_u^{u'} \mathrm{d}u''\, \mathcal{F}_0(u'')$$

- For nth moment use $\tilde{Q}_n$ (flux has old and new terms)

$$\begin{aligned}\frac{1}{4}\mathcal{D}_n\cdots\mathcal{D}_1\mathscr{D}^i\mathscr{D}^j \underset{(n)}{\mathcal{N}}_{ij}(u',u) = {}& \tilde{Q}_n(u';u') - \tilde{Q}_n(u;u') \\ &+ \int_u^{u'} \mathrm{d}u''\, \Big[\mathcal{F}_n(u'') + \overbrace{(\cdots)}^{\text{constructed from } \mathcal{G}_n}\Big] \\ &+ \int_u^{u'} \mathrm{d}u''\, \underbrace{(\cdots)}_{\text{integrals from lower orders}}\end{aligned}$$

Example: drift memory/first moment

- Modified charge:

$$\tilde{Q}_1(u;\tilde{u}) \equiv Q_1(u) + (\tilde{u}-u)\mathcal{D}_1 Q_0(u)$$

- Expression for "electric" first moment, the c.o.m. memory [Nichols, 1807.08767]

$$\frac{1}{4}\mathcal{D}_1 \mathscr{D}^i \mathscr{D}^j \underset{(1)}{\mathcal{N}}_{ij}(u',u) = \tilde{Q}_1(u';u') - \tilde{Q}_1(u;u') + \int_u^{u'} \mathrm{d}u\, \mathcal{F}_1(u'') + \int_u^{u'} \mathrm{d}u'' \int_u^{u''} \mathrm{d}u'''\, \mathcal{D}_1 \mathcal{F}_0(u''')$$

- Spin memory: "magnetic" first moment [Pasterski et al., 1502.06120]

$$\frac{1}{4}\mathcal{D}_1 \mathscr{D}^i \mathscr{D}^j ({}^*\underset{(1)}{\mathcal{N}})_{ij}(u',u) = \tilde{Q}_1^*(u';u') - \tilde{Q}_1^*(u;u') + \int_u^{u'} \mathrm{d}u\, \mathcal{F}_1^*(u'')$$

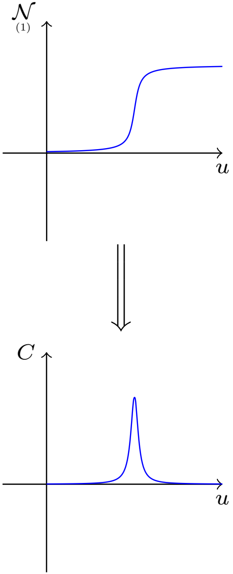
Contribution to shear

- Current detectors measure *shear*, not moments of news!
- From moments, can recover shear (up to constant):

$$C_{ij}(u') - C_{ij}(u) = \frac{\partial^n}{\partial u'^n} \underset{(n)}{\mathcal{N}}_{ij}(u', u)$$

(note: only this simple for the Cauchy moments!)

- Previous slides: contributions to moments of news
  $\implies$ parts of shear arising from these contributions
- Nonlinear contributions from (e.g.) $\mathcal{F}_n$ give a signal that can be detected



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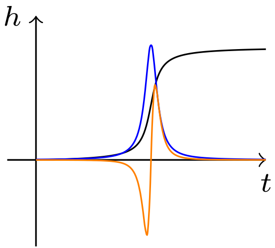
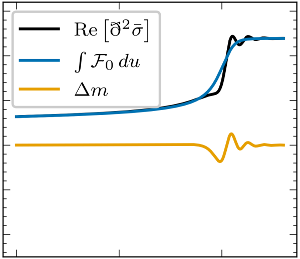
Numerical relativity [**Grant** & Mitman, 2312.02295]

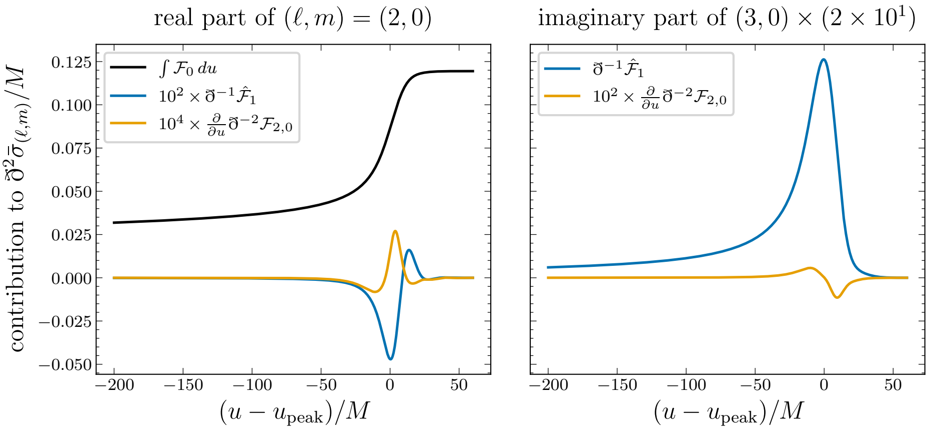
- Considered equal-mass, quasicircular binary
- With CCE, *full* waveform available
- Only $m = 0$, should be mostly non-oscillatory
- Can test:
  - Are shapes of the different flux contributions what we would expect?
  - Are the charges or fluxes more important?
- Note: to translate from Newman-Penrose:

$$\sigma \sim C_{ij}, \quad \psi_1 \sim Q_1, \quad \psi_0 \sim Q_2, \quad \eth \sim \mathscr{D}_i$$



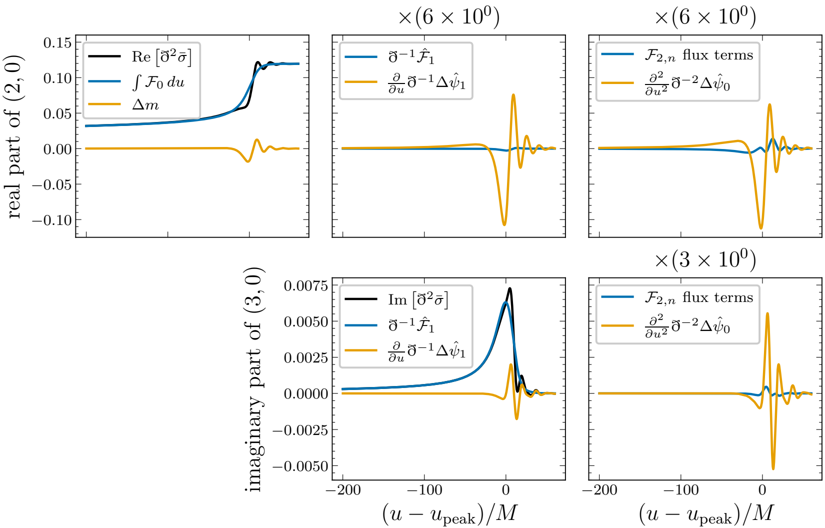


Shape during merger



- Some flux contributions have expected shapes, but not all
- Spoiled by ringdown?

Charge vs. flux



- ► Charge often larger than flux

Post-Newtonian theory [Siddhant, **Grant**, & Nichols; 2403.13907]

- Advantages/disadvantages relative to NR:
  - \+ Completely analytic
  - \+ Can consider wider set of parameters (unequal masses, etc.)
  - \+ PN scaling proxy for "detectability"
  - – Valid only for inspiral (memory mostly at merger)
  - – Charges harder to compute

- Also study $m \neq 0$, "oscillatory" memory
  - Not typically considered to be memories
  - Still sourced by flux/pseudo-flux terms as non-oscillatory effects

- "Alternative" way to understand nonlinearities in PN multipole moments:

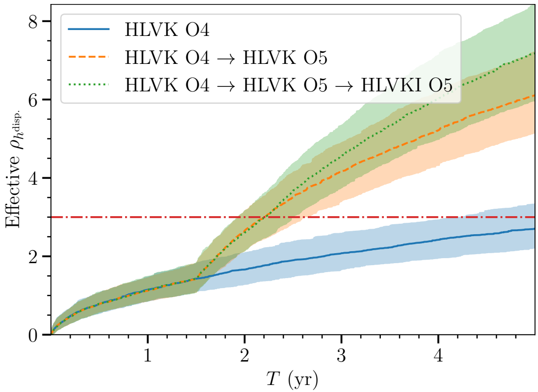
$$\overbrace{I_L, J_L}^{\text{“source”}} \implies \underbrace{\overbrace{M_L, S_L}^{\text{“canonical”}} \implies \overbrace{U_L, V_L}^{\text{“radiative”}}}_{\text{higher memories appear here}}$$

Post-Newtonian orders

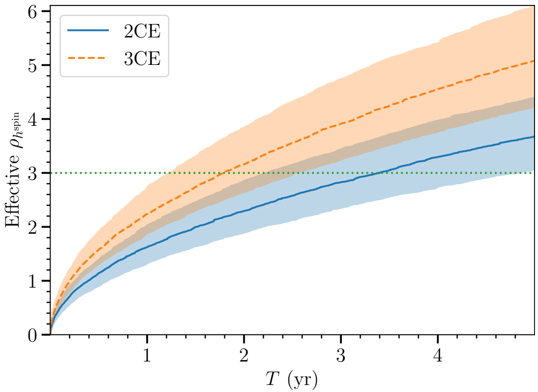
Flux or pseudo-flux	Non-oscillatory ($m = 0$) Leading modes	Non-oscillatory ($m = 0$) PN order	Oscillatory ($m \neq 0$) Leading modes	Oscillatory ($m \neq 0$) PN order
$\mathcal{F}_0$	$l = 2, 4$	0	$l = 4$ ($m = \pm 4$)	2.5
$\mathcal{F}_1$	$l = 2, 4$	5	$l = 3, 5, 7$ ($m = \dots$)	3
$\mathcal{F}_1^*$	$l = 3$	2.5	$l = 3$ ($m = \pm 2$)	2.5
$\mathcal{F}_2$	$l = 2, 4, 6, 8$	10	$l = 2, 4, 8$ ($m = \dots$)	5
$\mathcal{F}_2^*$	$l = 3, 5, 7$	10	$l = 3, 5, 7$ ($m = \dots$)	5
$\mathcal{G}_2$	$l = 2, 4$	4	$l = 2$ ($m = \pm 2$)	1.5
$\mathcal{G}_2^*$	$l = 3$	6.5	$l = 2$ ($m = \pm 1$)	2

- Except for displacement & spin, oscillatory effects *far* lower order
- Some effects have been considered for detectability, others plausible?

The one slide on detectability [**Grant** & Nichols, 2210.16266]



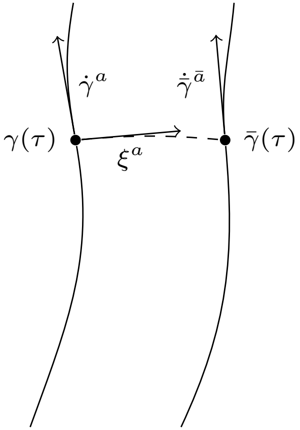
Displacement memory ($\mathcal{F}_0$) in "current" detectors



Spin memory ($\mathcal{F}_1^*$) in Cosmic Explorer

Conclusions

- Higher memories: more general effects that idealized observers can measure
- Like the usual (displacement) memory, they
  - Probe nonlinearities in GW propagation
  - (Can) arise as non-oscillatory parts of the GW signal
- Numerical & post-Newtonian binary inspirals:
  - Suspicion confirmed that these effects are small
  - Cosmic Explorer may see leading, “spin” memory



Future work

- Other theories in which to consider these effects:
  - Electromagnetism, classical Yang-Mills?
  - Modified gravity: can they tell us something normal memory cannot?
- Will some of these effects be detectable?
  - Non-oscillatory effects very small
  - Oscillatory effects need to be dug out of much larger oscillatory signal?